NEODREĐENI INTEGRAL

Ako je f(x) neprekidna funkcija i **F `(x) = f(x)** onda je $\boxed{\int f(x)dx = F(x) + C}$, gde je C proizvoljna konstanta.

Naučiti tablicu osnovnih integrala:

1. $\int dx = x + C$

2. $\int xdx = \frac{x^2}{2} + C$

3. $\boxed{\int x^n dx = \frac{x^{n+1}}{n+1} + C}$ najčešće se koristi...

4. $\int \frac{1}{x} dx = \ln|x| + C$ ili da vas ne zbuni $\int \frac{dx}{x} = \ln|x| + C$

5. $\int a^x dx = \frac{a^x}{\ln a} + C$

6. $\int e^x dx = e^x + C$

7. $\int \sin xdx = -\cos x + C$

8. $\int \cos xdx = \sin x + C$

9. $\int \frac{1}{\sin^2 x} dx = -ctgx + C$

10. $\int \frac{1}{\cos^2 x} dx = tgx + C$

11. $\int \frac{1}{1+x^2} dx = \begin{matrix} arctgx + C \quad \text{ili} \\ -arcctgx + C \end{matrix}$ to jest $\boxed{\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} arctg \frac{x}{a} + C}$

12. $\int \frac{1}{\sqrt{1-x^2}} dx = \begin{matrix} \arcsin x + C \quad \text{ili} \\ -arccocx + C \end{matrix}$ to jest $\boxed{\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin \frac{x}{a} + C}$

Primjeri:

1. $\int x^5 dx = \frac{x^{5+1}}{5+1} + C = \frac{x^6}{6} + C$ kao 3. tablični $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

2. $\int 7^x dx = \frac{7^x}{\ln 7} + C$ kao 5. tablični $\int a^x dx = \frac{a^x}{\ln a} + C$

3. $\int \sqrt{x} dx = \int x^{\frac{1}{2}} dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2x^{\frac{3}{2}}}{3} + C$

4. $\int \frac{1}{x^{12}} dx = \int x^{-12} dx = \frac{x^{- \quad +}}{-12+1} + C = \frac{x^{-}}{-11} + C = \frac{1}{-11x^{11}} + C$

5. $\int x \cdot \sqrt[3]{x} dx =$ Upotrijebimo pravila za stepen i korjen

$x \cdot \sqrt[3]{x} = x^1 \cdot x^{\frac{1}{3}} = x^{1+\frac{1}{3}} = x^{\frac{4}{3}}$

$\int x \cdot \sqrt[3]{x} dx = \int x^{\frac{4}{3}} dx = \frac{x^{\frac{4}{3}+1}}{\frac{4}{3}+1} + C = \frac{x^{\frac{7}{3}}}{\frac{7}{3}} + C = \frac{3x^{\frac{7}{3}}}{7} + C$

6.

$\int \sqrt{x\sqrt{x\sqrt{x}}} dx = ?$

$\sqrt{x\sqrt{x\sqrt{x}}} = \sqrt{x\sqrt{\sqrt{x^2 \cdot x}}} = \sqrt{x\sqrt[4]{x^3}} = \sqrt{\sqrt[4]{x^4 \cdot x^3}} = \sqrt[8]{x^7} = x^{\frac{7}{8}}$

$\int x^{\frac{7}{8}} dx = \frac{x^{\frac{7}{8}+1}}{\frac{7}{8}+1} + C = \frac{x^{\frac{15}{8}}}{\frac{15}{8}} + C = \frac{8x^{\frac{15}{8}}}{15} + C$

osnovna svojstva neodredjenog integrala:

1) $\boxed{\int A \cdot f(x) dx = A \cdot \int f(x) dx}$ gde je A konstanta (broj)

Dakle, slično kao i kod izvoda, konstanta (broj) izlazi ispred integrala...

Primjeri:

8.

$$\int 4x^3dx = ?$$

$$\int 4x^3dx = 4\int x^3dx = 4\frac{x^4}{4} + C = x^4 + C$$

9.

$$\int \frac{1}{4x}dx = ?$$

$$\int \frac{1}{4x}dx = \frac{1}{4}\cdot\int \frac{1}{x}dx = \frac{1}{4}\cdot\ln|x| + C$$

10.

$$\int 2\pi \sin x dx = ?$$

$$\int 2\pi \sin x dx = 2\pi\cdot\int \sin x dx = 2\pi\cdot(-\cos x) + C = -2\pi\cos x + C$$

2) $\boxed{\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx}$

Opet slično kao kod izvoda: Ako imamo zbir ili razliku više funkcija od svake tražimo posebno integral...

11.

$$\int (4x^2 + 2x - 3)dx = ?$$

$\int (4x^2 + 2x - 3)dx = \int 4x^2dx + \int 2xdx - \int 3dx =$ konstante izbacimo ispred integrala...

$$= 4\int x^2dx + 2\int xdx - 3\int dx$$

$$= 4\frac{x^3}{3} + 2\frac{x^2}{2} - 3x + C = \boxed{4\frac{x^3}{3} + x^2 - 3x + C}$$

12.

$$\int (5\cos x + \frac{1}{3}e^x - 2x^3 + \frac{4}{x} - \frac{23}{\sin^2 x} + 2\cdot 5^x)dx = ?$$

$$\int (5\cos x + \frac{1}{3}e^x - 2x^3 + \frac{4}{x} - \frac{23}{\sin^2 x} + 2\cdot 5^x)dx = 5\int \cos x dx + \frac{1}{3}\int e^x dx - 2\int x^3dx + 4\int \frac{1}{x}dx - 23\int \frac{dx}{\sin^2 x} + 2\cdot\int 5^x dx =$$

$$= 5\sin x + \frac{1}{3}e^x - 2\frac{x^4}{4} + 4\ln|x| - 23(-ctgx) + 2\frac{5^x}{\ln 5} + C$$

13.

$$\int \frac{x-2}{x^3}dx = ?$$

Kod ovog i sličnih integrala ćemo upotrijebiti $\boxed{\frac{A \pm B}{C} = \frac{A}{C} \pm \frac{B}{C}}$

$$\int \frac{x-2}{x^3}dx = \int (\frac{x}{x^3} - \frac{2}{x^3})dx = \int (x^{-2} - 2x^{-3})dx = \int x^{-2}dx - 2\int x^{-3}dx$$

$$= \frac{x^{-2+1}}{-2+1} - 2\frac{x^{-3+1}}{-3+1} + C$$

$$= \boxed{-\frac{1}{x} + \frac{1}{x^2} + C}$$

14.

$$\int \frac{x^2}{x^2+1}dx = ?$$

$$\int \frac{x^2}{x^2+1}dx = \int \frac{x^2+1-1}{x^2+1}dx = \int \frac{x^2+1}{x^2+1}dx - \int \frac{1}{x^2+1}dx = \int \frac{\cancel{x^2+1}}{\cancel{x^2+1}}dx - \int \frac{1}{x^2+1}dx$$

$$= \int dx - \int \frac{1}{x^2+1}dx = \boxed{x - arctgx + C}$$

15.

$$\int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x}dx = ? \qquad \text{treba nam formula } \boxed{\cos 2x = \cos^2 x - \sin^2 x}$$

$$\int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x}dx = \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x \cdot \cos^2 x}dx = \quad \text{rastavimo na dva integrala...}$$

$$\int \frac{\cos^2 x}{\sin^2 x \cdot \cos^2 x}dx - \int \frac{\sin^2 x}{\sin^2 x \cdot \cos^2 x}dx = skratimo...$$

$$\int \frac{\cancel{\cos^2 x}}{\sin^2 x \cdot \cancel{\cos^2 x}}dx - \int \frac{\cancel{\sin^2 x}}{\cancel{\sin^2 x} \cdot \cos^2 x}dx = \text{ i dobijamo dva tablična integrala...}$$

$$\int \frac{1}{\sin^2 x}dx - \int \frac{1}{\cos^2 x}dx = \boxed{-ctgx - tgx + C}$$

16.

$$\int \frac{dx}{\sin^2 x \cdot \cos^2 x} = ? \qquad \text{treba nam formula } \boxed{\sin^2 x + \cos^2 x = 1}$$

$$\int \frac{1 \cdot dx}{\sin^2 x \cdot \cos^2 x} = \text{uz dx je 1, zar ne?} = \int \frac{1}{\sin^2 x \cdot \cos^2 x}dx = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cdot \cos^2 x}dx = \quad \text{rastavimo na dva integrala...}$$

$$\int \frac{\sin^2 x}{\sin^2 x \cdot \cos^2 x}dx + \int \frac{\cos^2 x}{\sin^2 x \cdot \cos^2 x}dx = skratimo...$$

$$\int \frac{\cancel{\sin^2 x}}{\cancel{\sin^2 x} \cdot \cos^2 x}dx + \int \frac{\cancel{\cos^2 x}}{\sin^2 x \cdot \cancel{\cos^2 x}}dx = \text{ i dobijamo dva tablična integrala...}$$

$$\int \frac{1}{\cos^2 x}dx + \int \frac{1}{\sin^2 x}dx = \boxed{tgx - ctgx + C}$$

17.

$$\int tg^2 x dx = ?$$

Ovde koristimo $tgx = \frac{\sin x}{\cos x}$

$$\int tg^2 x dx = \int \frac{\sin^2 x}{\cos^2 x}dx = \text{ kako je } \sin^2 \alpha + \cos^2 \alpha = 1 \to \boxed{\sin^2 \alpha = 1 - \cos^2 \alpha}\text{, pa je}$$

$$= \int \frac{1 - \cos^2 \alpha}{\cos^2 x}dx = \int \frac{1}{\cos^2 x}dx - \int \frac{\cancel{\cos^2 \alpha}}{\cancel{\cos^2 x}}dx = \int \frac{1}{\cos^2 x}dx - \int dx = \boxed{tgx - x + C}$$

19.

$$\int \frac{18x^2-2}{3x-1}dx = ?$$

Probamo da sredimo podintegralnu funkciju, ako neće, mora se koristiti neki drugi trik(druga metoda)...

$$\frac{18x^2-2}{3x-1} = \frac{2(9x^2-1)}{3x-1} = \frac{2\,\cancel{(3x-1)}\,(3x+1)}{\cancel{3x-1}} = 2(3x+1) = 6x+2$$ Sad je već lakše...

$$\int \frac{18x^2-2}{3x-1}dx = \int (6x+2)dx = 6\int xdx + 2\int dx = 6\frac{x^2}{2} + 2x + C = \boxed{3x^2+2x+C}$$

20.

$$\int \frac{4-x}{2+\sqrt{x}}dx = ?$$

$$\frac{4-x}{2+\sqrt{x}} = \frac{2^2-(\sqrt{x})^2}{2+\sqrt{x}} = \frac{(2-\sqrt{x})(2+\sqrt{x})}{2+\sqrt{x}} = \frac{(2-\sqrt{x})\,\cancel{(2+\sqrt{x})}}{\cancel{2+\sqrt{x}}} = 2-\sqrt{x} = 2-x^{\frac{1}{2}}$$

$$\int \frac{4-x}{2+\sqrt{x}}dx = \int (2-x^{\frac{1}{2}})dx = \int 2dx - \int x^{\frac{1}{2}}dx = 2x - \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C = \boxed{2x - \frac{2x^{\frac{3}{2}}}{3} + C}$$