Priprema za pismeni

1. Odrediti ostatak pri djeljenju polinoma:
   a) $A(x) = x^4 - 2x^2 + x - 10 \; sa \;\; x - 2$
   b) $A(x) = 5x^3 - 4x + 7 \; sa \; x - 3$
   c) $A(x) = \; -7x^2 + 2x - 1 \; sa \; x + 1$
   d) $A(x) = 2x^4 - 3x^2 + x \;\; sa \;\; x + 2$

   R: Bezuov stav: Ostatak pri djeljenju polinoma A(x) sa polinomom x-a je A(a).

   a) $x - 2 \; \rightarrow a = 2$. *Ukoliko niste sigurni, primijenite postupak:* $x - 2 = 0 \; \rightarrow x = 2$
   Ostatak: $A(2) = 2^4 - 2 \cdot 2^2 + 2 - 10 = 16 - 8 - 8 = 0$

   b) $x - 3 \; \rightarrow a = 3$
   Ostatak: $A(3) = 5 \cdot 3^3 - 4 \cdot 3 + 7 = 135 - 5 = 130$

   c) $x + 1 \; \rightarrow a = -1 \; (x + 1 = 0 \; \rightarrow x = -1)$
   Ostatak: $A(-1) = -7 \cdot (-1)^2 + 2(-1) - 1 = -10$

2. Rastaviti na činioce polinom:
   a) $A(x) = \; x^2 - 4$
   b) $A(x) = \; x^2 - 10x + 25$
   c) $A(x) = \; x^2y - x^3$
   d) $A(x) = 6x^2 - 24$
   e) $A(x) = \; x^2 - 8x + 12$
   f) $A(x) = x^4 - 4x^2 + x + 2$
   g) $A(x) = 2x^2 - 3x - 5$

   R: a) $A(x) = \; x^2 - 4 = (x - 2)(x + 2)$

   b) $A(x) = \; x^2 - 10x + 25 = \; x^2 - 2 \cdot x \cdot 5 + 25 = (x - 5)^2$

   c) $A(x) = \; x^2y - x^3 = x^2(y - x)$

   d) $A(x) = 6x^2 - 24 = 6(x^2 - 4) = 6(x - 2)(x + 2)$

e) dopuna do kvadrata razlike

f) $A(x) = x^4 - 4x^2 + x + 2 = x^2(x^2 - 4) + x + 2 = x^2(x-2)(x+2) + (x+2) = (x+2)(x^2(x-2)+1) = (x+2)(x^3 - 2x^2 + 1)$

3. Koristeći Bezuov stav, rastaviti polinom na proste činioce:
   a) $A(x) = x^3 + 3x^2 - 4x - 12$
   b) $A(x) = x^3 - 2x^2 - 5x + 6$
   c) $A(x) = x^3 - 6x^2 + 11x - 6$

4. Odrediti oblast definisanosti sljedećih algebarskih izraza:
   a) $\frac{2}{x+5}$
   b) $\frac{3-x}{2x+5}$
   c) $\sqrt{x-4}$
   d) $\sqrt{8 - \frac{2}{3}x}$
   e) $\frac{1}{x^2-3x+2} + 3x$
   f) $\frac{4}{x^2-9} + \sqrt{2x-3}$
   g) $\frac{2-x}{\sqrt{3x-5}}$

R: a) Uslov: $x + 5 \neq 0 \rightarrow x \neq -5$

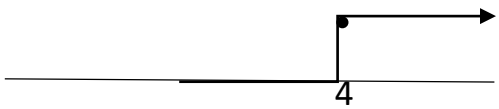
Oblast definisanosti: $x \in (-\infty, -5) \cup (-5, +\infty) = R \setminus \{-5\}$

b) Uslov: $2x + 5 \neq 0 \rightarrow 2x \neq -5 \rightarrow x \neq -\frac{5}{2}$

Oblast definisanosti: $x \in \left(-\infty, -\frac{5}{2}\right) \cup \left(-\frac{5}{2}, +\infty\right) = R \setminus \left\{-\frac{5}{2}\right\}$

c)Uslov: $x - 4 \geq 0 \rightarrow x \geq 4$

Oblast definisanosti: $x \in [4, +\infty)$



e) $\frac{1}{x^2-3x+2} + 3x$ (postoji problem samo u prvom sabirku)

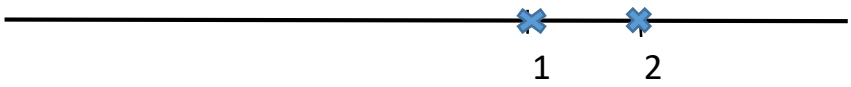
Uslov: $x^2 - 3x + 2 \neq 0$ (Moramo rastaviti ovaj polinom na proste činioce)

$$x^2 - 3x + 2 = x^2 - 2x - x + 2 = x(x-2) - (x-2) = (x-2)(x-1)$$

$$\rightarrow (x-2)(x-1) \neq 0$$

$$x - 2 \neq 0 \bigwedge x - 1 \neq 0$$

$$x \neq 2 \; i \; x \neq 1$$



Oblast definisanosti: $x \in (-\infty, 1) \cup (1,2) \cup (2, +\infty) = R \setminus \{1,2\}$

g) $\frac{2-x}{\sqrt{3x-5}}$

Uslov: $3x - 5 \geq 0$ i $3x - 5 \neq 0$ (u ovom članu imamo dva uslova, jer imamo razlomak i u imeniocu imamo i kvadratni korijen).

$$\Longrightarrow 3x - 5 > 0$$

5. Uprostiti algebarski izraz i napisati pod kojim uslovima važi transformacija:

a) $\frac{x}{x^2+10x+25} - \frac{1}{x+5} + \frac{x}{x-3}$

b) $\frac{a}{a-2} + \frac{3}{a+3}$

c) $\frac{x-1}{x+1} - \frac{2}{x^2-1}$

d) $\left(\frac{x}{y^2+xy}+\frac{2}{x+y}+\frac{y}{x^2+xy}\right):\frac{x}{y}$

e) $\left(2+\frac{2x^2}{2+x}-x\right)\cdot\frac{2+x}{x}$

f) $\frac{a+1}{a+2}+\frac{6a}{a^2-4}-\frac{2a-1}{a-2}$