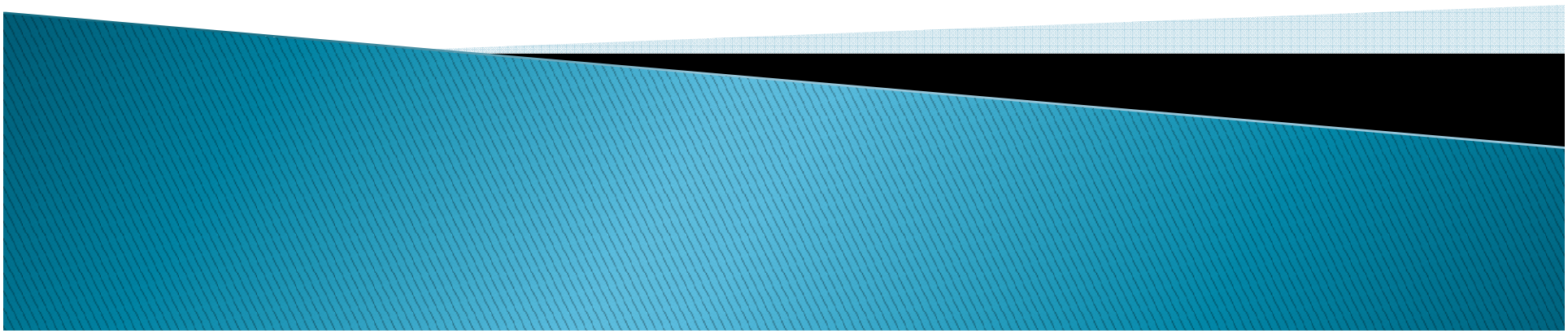
Monotonost funkcije



MONOTONOST FUNKCIJE

Neka je funkcija f(x) diferencijabilna na intervalu (a,b) tj. ima izvod u svakoj tački toga intervala.

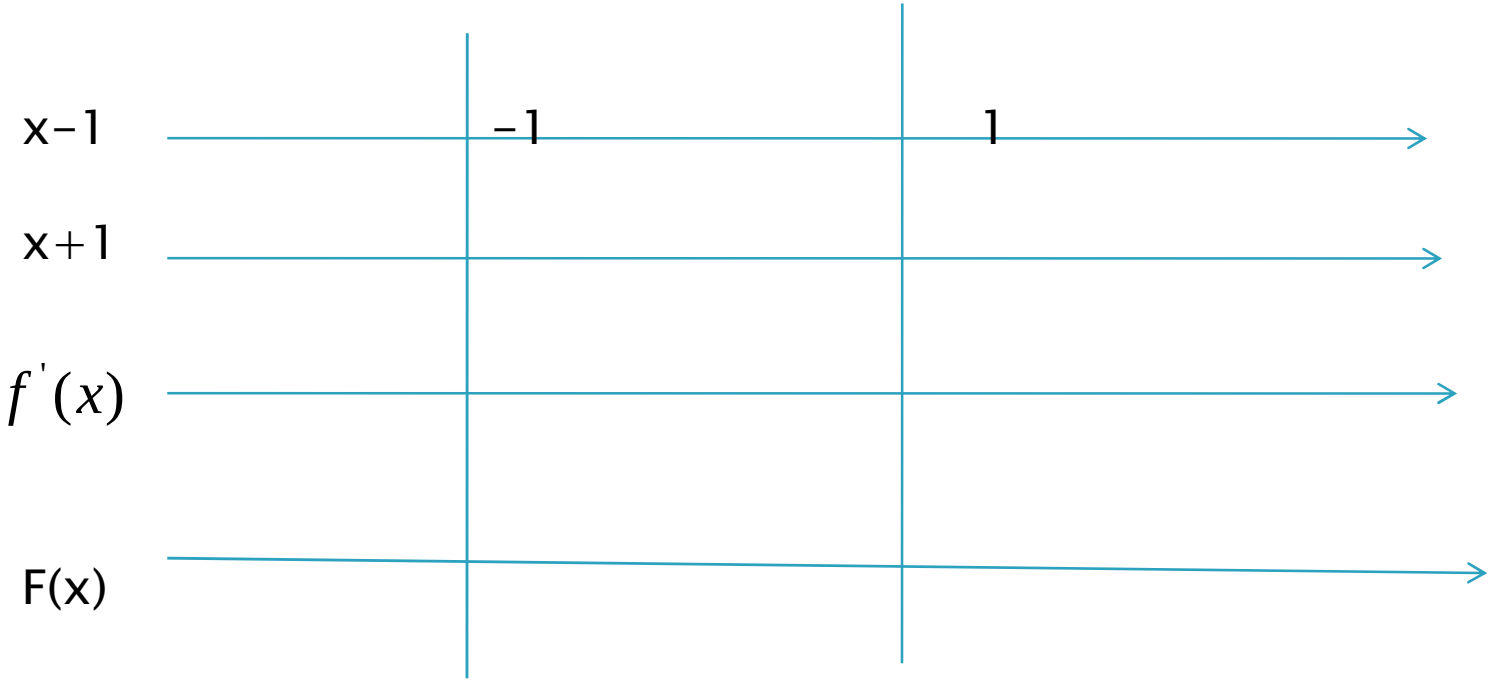
1. Ako je $f'(x) > 0 \ \forall x \in (a,b)$ tada je funkcija f(x**) rastuća** na intervalu (a,b)

2. Ako je $f'(x) < 0 \ \forall x \in (a,b)$ tada je funkcija f(x) **opadajuća** na intervalu (a,b)

Primjer (osnovni): Odrediti intervale monotonosti funkcije

$$f(x)=x^3-3x$$

$$f'(x)=3x^2-3=3(x^2-1)=3(x-1)(x+1)$$



Zadaci :

1. Odrediti intervale monotonosti funkcije

$$y = -x^3 - 4x^2 - 4x$$

$$y' = -3x^2 - 8x - 4,\ y' = 0 \Leftrightarrow -3x^2 - 8x - 4 = 0$$

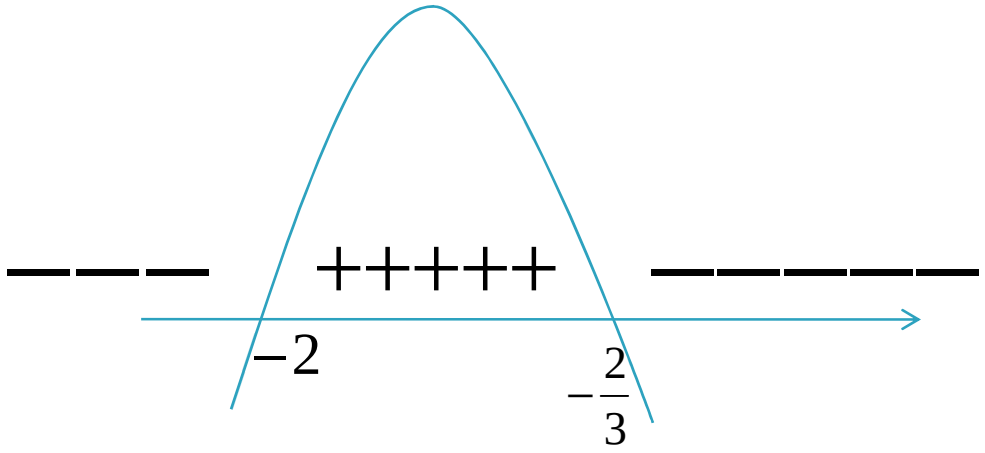
$$a = -3$$

$$b = -8$$

$$c = -4$$

$$x_{1,2} = \frac{8 \pm \sqrt{64 - 4 \cdot (-3) \cdot (-4)}}{2 \cdot (-3)}$$

$$x_{1,2} = \frac{8 \pm \sqrt{16}}{-6} = \frac{8 \pm 4}{-6} \Rightarrow x_1 = -2,\ x_2 = -\frac{2}{3}$$



$y' > 0 \, za \, x \in (-2, -\frac{2}{3}) \Rightarrow$ f rastuća

$y' < 0 \, za \, x \in (-\infty, -2) \cup (-\frac{2}{3}, +\infty) \Rightarrow$ f opadajuća

2.

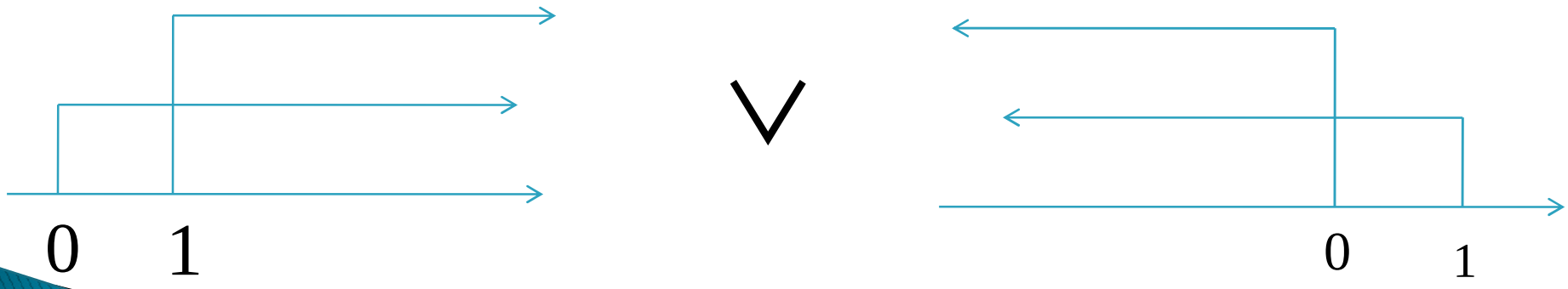
$$y = x - \ln x$$

$$y' = 1 - \frac{1}{x} = \frac{x-1}{x}, \; x \neq 0$$

$$\frac{A}{B} > 0 \Leftrightarrow (A > 0 \wedge B > 0) \vee (A < 0 \wedge B < 0)$$

$$(x - 1 > 0 \wedge x > 0) \vee (x - 1 < 0 \wedge x < 0)$$

$$(x > 1 \wedge x > 0) \vee (x < 1 \wedge x < 0)$$



x	$(-\infty,0)$	0	$(0,1)$	1	$(1,+\infty)$
$x-1$	$-$		$-$		$+$
x	$-$		$+$		$+$
y'	$+$		$-$		$+$

Zadaci za vježbanje :

1.Odrediti intervale monotonosti datih funkcija

$$a)\, y = \frac{1}{3}x^3 - \frac{3}{2}x^2 - 4x$$

$$b)\, y = x^2 - \ln x^2$$